GEORGIA INSTITUTE OF TECHNOLOGY — OFFICE OF CONTRACT ADMINISTRATION

PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. E-20-697 — DATE 9/10/81

Project Director: Dr. S.N. Atluri — School/~~Lab~~ Civil Engineering

Sponsor: Office of Naval Research, Arlington

Type Agreement: Contract No. N00014-78-C-0636

Award Period: From 9/1/81 To ~~8/31/83~~ 8/31/85 (Performance) ~~10/31/83~~ 10/31/85 (Reports)

Sponsor Amount: $120,000 (incrementally funded at $55,000 thru 9/1/82)* — Contracted through:

Cost Sharing: $26,561 (E-20-360) incrental at $11,366 — GTRI/~~GIT~~

Title: Non-Standard Finite Element Method in Nonlinear, Inelastic, Solid Mechanics: Fracture and Integrity Analysis

ADMINISTRATIVE DATA

OCA Contact Faith Costello

1) Sponsor Technical Contact:

(Scientific Officer)
Director, Material Sciences Division
Structural Mechanics
800 N. Quincy Street
Department of the Navy
Arlington, VA 22217

2) Sponsor Admin/Contractual Matters:

Mr. Thomas A. Bryant
ONR RR
214 O'Keefe Bldg.
Atlanta, GA 30332

Defense Priority Rating: DO-C9 under DMS Reg 1 — Security Classification: NONE

RESTRICTIONS

See Attached Gov't Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval — Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of $500 or 125% of approved proposal budget category.

Equipment: Title vests with Government, except that items with a cost of less than $1,000 vest with GIT upon acquisition if prior approval to purchase obtained from Contracting Officer

COMMENTS:

*Contract total amount based on funding effective with Mod. No. P00003. Actual total amount of contract is $244,441.

-Continuation of project no. E-20-668

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GEORGIA INSTITUTE OF TECHNOLOGY | OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 5/15/86

Project No. E-20-697 School/XXX CE

Includes Subproject No.(s) N/A

Project Director(s) S. N. Atluri GTRI/XXX

Sponsor Office of Naval Research

Title - Non-Standard Finite Element Method in Nonlinear, Inelastic, Solid Mechanics: Fracture and Integrity Analysis

Effective Completion Date: 8/31/85 (Performance) 10/31/85 (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [ ] Final Invoice or Final Fiscal Report - already submitted.
- [x] Closing Documents
- [x] Final Report of Inventions
- [x] Govt. Property Inventory & Related Certificate
- [ ] Classified Material Certificate
- [ ] Other

Continues Project No. E-20-668

Continued by Project No.

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E-26-...

Preface:

In the following, a research proposal in support of the request for renewal of funding under the present ONR Contract No. N00014-78-C-0636, for a period of 27 months, between 9/1/85 to 12/1/87 is presented. The proposed research will be continued at the Center for the Advancement of Computational Mechanics of Georgia Institute of Technology with S.N. Atluri as the principal investigator.

The technical progress during the current contract period is first briefly reviewed, following which, brief discussions of proposed research and planned approaches are given.

I. Technical Progress During the Current Contract Period:

A number of significant results in elastic-plastic and dynamic fracture mechanics have been obtained during the course of the current research. A few are briefly described below while a complete list of publications resulting from this research, which document in detail all the research accomplished, is given at the end of this section.

The T^* parameter (and its incremental counterpart ΔT^*) has been established to be of relevance in the mechanics of elastic-plastic fracture. Specifically it has been shown to have several advantages over the currently widely used fracture parameters such as J and CTOA. Finite element analyses of experimental data on elastic-plastic stable crack growth were carried out as a means to obtain a comparison of the effectiveness of the plastic fracture parameters, and T^* was found to be clearly superior.

In current practice, the J-integral is commonly used to determine the resistance of plastically deformed structures to continued crack growth. This approach has been shown, during current research, to be valid only for monotonic loading and small amounts of crack growth; yet in the engineering community the J-resistance curve approach is believed to be conservative for all general loadings. On the other hand, the T* parameter is formulated to be theoretically valid under a wide range of loading/unloading conditions. These two fracture criteria (T* and J), along with CTOA (crack-tip opening angle), were examined for their validity and predictive capability in situations of crack growth after a cycle of loading, unloading to zero load, followed by reloading. The results of a careful, combined numerical/experimental study showed that the T* parameter accurately predicted the behavior (crack growth to begin only after 50% loading in the reloading phase), while the other parameters (J and CTOA) were seriously anti-conservative. In another study, the relevance of the parameters $\dot{T}^*$, C^*, and $\dot{T}_c$ in characterizing creep crack growth was examined. Experimental data on creep crack growth in a 316 stainless steel single-edge notch specimen was numerically simulated, and the variations of various parameters during crack growth were ascertained. The results were found conclusively in favor of the $\dot{T}^*$ parameter in characterizing creep crack growth under non-steady creep (not pure power law creep) as well as in situations wherein time-independent plastic strains are significant in addition to creep strains. These results are also significant in the contest of current and continued studies on crack propagation in viscoplastic materials.

Comprehensive studies have been conducted concerning (i) possible crack-tip parameters that may govern elasto-dyamic crack propagation in the presence of non-uniform temperature fields (caused by high-intensity energy sources), material inhomogeneity, etc.; (ii) crack-tip parameters in elasto-plastic dynamic fracture; and (iii) crack-tip parameters in rate-sensitive elastic-plastic solids. In this process, several parameters $\tilde{J}'$ (elasto-dynamic crack propagation), ΔT^* (incremental parameters governing crack propagation in rate-independent or rate-dependent elasto-plastic solids), have been established to be theoretically valid.

The path-independent integral J'_k, which has the meaning of energy release rate in elasto-dynamic crack propagation, has been used numerically to obtain the mixed mode dynamic stress-intensity factors for a crack propagating in a prescribed direction with a prescribed velocity. Moving isoparametric (non-singular) elements are used to model crack propagation. Even though $\underline{J}'$ is a vector integral and hence is coordinate invariant, the desirability of using specific coordinate systems to improve the accuracies of the numerical solutions for K-factors is pointed out. Two procedures for extracting the mixed mode K-factors from the $\underline{J}'$ integral for a propagating crack are given. It is found that the component of $\underline{J}'$ along the crack axis, i.e. J'^{o} is always equal to or greater than the product of a crack-velocity function and the component normal to the crack-axis, J'^{o}. Comprehensive results were obtained to validate these procedures for engineering applications.

A fundamental study concerning the adiabatic heating due to plastic energy dissipation near a rapidly propagating crack in structural steel has been conducted. The transient temperature field (as seen by a moving observer) due to heat generated in the moving process-zone near a dynamically propagating crack-tip was analyzed by a moving-mesh finite element procedure. The effects of temperature-dependent material properties, and the loss of heat to the surrounding medium through convection and radiation are studied. Situations under which conditions in the process zone may be labelled as 'isothermal' or 'adiabatic' have been clearly delineated. Precise estimates of temperature rise near the tips of cracks propagating in structural steels have been obtained. The effects of this temperature rise on the process of fracture itself is a subject of continued inquiry.

To account for the phenomena of unloading near a propagating crack-tip, and to account for material behavior near the crack-tip under arbitrary far-field loading histories (of loading/unloading/reloading, etc.), the present computational procedures have been systematically enhanced with more realistic constitutive relations. As by-products of these efforts, certain unifying concepts underlying the internal-time, general internal variable, and multi-yield-surface theories of plasticity have been found. Rate forms of evolution equations for stress, back stress, and other variables based on a new internal-time theory have been developed. It has been shown that by an appropriate choice of kernels in the presently developed theory, the so-called "nonlinear-kinematic-hardening" rules of Chaboche, Mroz, et al., the multiple-yield-surface theories of Mroz, Krieg, Dafalias, etc., and the linear kinematic hardening theories of Prager, etc., may all be deduced as special cases. Computational algorithms for the implementation of this new theory have been systematically implemented. It has been demonstrated that while the present theory is no more difficult to implement computationally than the usual Prager-Ziegler theories, it leads to far better constitutive modeling of observed phenomenological behavior in plasticity.

Computational algorithms for numerical modeling of elastic as well as inelastic dynamic fracture have been comprehensively studied. Node-release techniques as well as mesh-translation techniques and time-integration schemes have been analyzed as to convergence and stability. Especially for rate-sensitive plastic behavior, the limits on the time-step in the time integration procedure, as dictated by the constitutive model itself, the stability of the time integration and the required 'smoothness' of the node-release (or equivalently the mesh-shift) have been systematically explored.

While the above represents a brief summary of some of the salient findings of the current research, a comprehensive discussion of the above as well as other findings of research is given in the following publications in the archival literature:

1. S.N. Atluri, M. Nakagaki, T. Nishioka, and Z-b. Kuang, "Crack-Tip Parameters and Temperature Rise in Dynamic Crack Propagation", Engineering Fracture Mechanics (Special Issue in Honor of A.S. Kobayashi; Eds: M.F. Kanninen, S.N. Atluri, Y. Rajapakse, and W.G. Knauss) (In Press).

2. F.W. Brust, J.J. McGowan, and S.N. Atluri, "A Combined Numerical/Experimental Study of Ductile Crack Growth After a Large Unloading, Using T*, J, and CTOA Approaches", Engineering Fracture Mechanics (In Press).

3. F.W. Brust, T. Nishioka, and S.N. Atluri, "Further Studies on Elastic-Plastic Stable Fracture Utilizing the T* Integral", Engineering Fracture Mechanics (In Press).

4. F.W. Brust and S.N. Atluri, "Studies on Creep Crack Growth Using the T* Integral", Engineering Fracture Mechanics (In Press).

5. Z-b. Kuang and S.N. Atluri, "Temperature Field Due to a Moving Heat Source: A Moving Mesh Finite Element Analysis", Journal of Applied Mechanics, Paper No. 85-APM-19, ASME (In Press).

6. S.N. Atluri and T. Nishioka, "Numerical Studies in Dynamic Fracture Mechanics", International Journal of Fracture (Special 20th anniversary Volume Issue on Dynamic Fracture) (In Press).

7. O. Watanabe and S.N. Atluri, "A New Endochronic Approach to Computational Elastoplasticity: Example of a Cyclically Loaded Cracked Plate", Journal of Applied Mechanics, ASME (In Press).

8. O. Watanabe and S.N. Atluri, "Internal Time, General Internal Variable and Multi-Yield-Surface Theories of Plasticity and Creep: A Unification of Concepts", International Journal of Plasticity, Pergamon Press (In Press).

9. S.N. Atluri, T. Nishioka, and M. Nakagaki, "Incremental Path-Independent Integrals in Inelastic and Dynamic Fracture Mechanics", Engineering Fracture Mechanics, Vol. 20, No. 2, pp. 209-244, 1984.

10. S.N. Atluri, T. Nishioka, and M. Nakagaki, "Recent Studies of Energy Integrals and Their Applications", Plenary Lecture, 6th International Congress on Fracture, New Delhi, India, 2-10 December 1984.

11. S.N. Atluri and T. Nishioka, "On Path-Independent Integrals in the Mechanics of Elastic, Inelastic, and Dynamic Fracture", Invited Paper, Special Issue of Honor of Professor George Irwin, Journal of Aeronautical Society of India, May 1984, 52 pp.

12. S.N. Atluri and T. Nishioka, "Computational and Theoretical Studies on Dynamic Fracture Mechanics and Three-Dimensional Crack Problems", Invited General Lecture and in Proceedings ICF International Symposium on Fracture Mechanics, Science Press, Beijing, December 1983, pp. 1-17.

13. T. Nishioka and s.N. Atluri, "On the Computation of Mixed-Mode K-Factors for a Dynamically Propagating Crack Using Path-Independent Integrals J'", Engineering Fracture Mechanics, Vol. 20, No. 2, pp. 193-208, 1984.

14. S.N. Atluri and A.S. Kobayashi, "Elastic-Plastic Fracture Mechanics", Chapter 3 in Computational Methods in the Mechanics of Fracture (Ed: S.N. Atluri), North-Holland Publishing Co. (To Appear).

15. S.N. Atluri, "Energetic Approaches and Path-Independent Integrals", Chapter 5 in Computational Methods in the Mechanics of Fracture (Ed: S.N. Atluri), North-Holland Publishing Co. (To Appear).

16. T. Nishioka and S.N. Atluri, "Computational Methods in the Mechanics of Dynamic Fracture", Chapter 8 in Computational Methods in the Mechanics of Fracture (Ed: S.N. Atluri), North-Holland Publishing Co. (To Appear).

17. S.N. Atluri, "Energy Release Rates and Path-Independent Integrals in Dynamic Fracture" in Material Behaviour Under High Stress and Ultrahigh Loading Rates (Eds: J. Mescall and V. Weiss), Plenum Press, NY, pp. 211-222, 1983.

18. S.N. Atluri, "Computational Solid Mechanics: Its Present Status and future Direction", General Lecture, 4th International Conference on Applied Mathematical Modeling, Tainan, Taiwan, December 1984.

19. M. Nakagaki, S.N. Atluri, and T. Nishioka, "On the Path-Independent Integral T_p in Elastic-Plastic Fracture", Developments in Mechanics, Vol. 12 (Proceedings SECTAM XII, Pine Mountain, GA), May 1984.

20. S.N. Atluri, "Crack-Tip Parameters in Elastic-Plastic and Dynamic Fracture", 2nd Navy Workshop on Fracture, Johns Hopkins University, Applied Physics Lab, May 1984.

21. T. Nishioka and s.N. Atluri, "On the Use of ($\underline{T}$) Integral in Visco-Plastic Dynamic Crack Propagation", Symposium on Recent Advances in Computational Mechanics, Joint ASME/ASCE Mechanics Conference, Albuquerque, NM, June 1985 (also to appear in Comp. Meth. Appl. Mech. & Engg.).

22. M. Nakagaki and S.N. Atluri, "Path-Independent Integrals Accounting for Thermomechanical Behavior Near Dynamically Propagating Cracks", Symposium on Recent Advances in Computational Mechanics, Joint ASME/ASCE Mechanics Conference, Albuquerque, NM, June 1985 (also to appear in Comp. Meth. Appl. Mech. & Engg.).

23. S.N. Atluri and T. Nishioka, "Energy Release Rates and Path-Invariant Integrals in Dynamic Fracture: Some Theoretical Developments and Computational Studies", Proceedings 4th International Conference on Fracture Mechanics in Engineering & Technology (Eds: G.C. Sih and R. Jones), Melbourne, 1982.

24. T. Nishioka and S.N. Atluri, "A Numerical Study of the Use of Path Independent Integrals in Elasto-Dynamic Crack Propagation", Engineering Fracture Mechanics, Vol. 18, No. 1, pp. 23-33, 1983.

25. T. Nishioka and S.N. Atluri, "Path-Independent Integrals, Energy Release Rates, and General Solutions of Near-Tip Fields in Mixed-Mode Dynamic Fracture Mechanics", Engineering Fracture Mechanics, Vol. 18, No. 1, pp. 1-22, 1983.

26. S.N. Atluri, "Current Studies in Inelastic, Dynamic, and Three-Dimensional Fracture Mechanics Analyses", Fracture Tolerance Evaluation (Eds: K. Kanazawa, A.S. Kobayashi, and K. Iida), Tokyo Printing Co., Japan, pp. 45-57, 1982.

27. T. Nishioka and S.N. Atluri, "Finite Element Simulations of Problems in Dynamic Fracture Mechanics", Transaction Japanese Society of Mechanical Engineers, JSME, pp. 138-145, 1982.

28. T. Niskioka and S.N. Atluri, "An Analysis of, and Some Observations on, Dynamic Fracture in an Impact Test Specimen,", Paper No. 81-PVP-18, presented at Joint Conference on Pressure Vessels and Piping, Materials, Nuclear Energy, and Solar Divisions, Denver, CO, June 1981; also in Journal of Pressure Vessel Technology.

29. T. Nishioka and S.N. Atluri, "A Method for Determining Dynamic Stress Intensity Factors from COD Measurement at the Notch Mouth in Dynamic Tear Testing", Engineering Fracture Mechanics, Vol. 16, No. 3, pp. 333-339, 1982.

30. T. Nishioka and S.N. Atluri, "Finite Element Simulation of Fast Fracture in Steel DCB Specimen", Engineering Fracture Mechanics, Vol. 16, No. 2, pp. 157-175, 1982.

31. T. Nishioka and S.N. Atluri, "Numerical Analysis of Dynamic Crack Propagation: Generation and Prediction Studies", Engineering Fracture Mechanics, Vol. 16, No. 3, pp. 303-332, 1982.

32. T. Nishioka, R.B. Stonesifer, and S.N. Atluri, "An Evaluation of Several Moving Singularity Finite Element Models for Fast Fracture Analyses", Engineering Fracture Mechanics, Vol. 15, No. 1-2, pp. 205-218, 1981.

33. S.N. Atluri and T. Nishioka, "Dynamic Fracture Analysis: A Translating Singularity Finite Element Procedure" in Advances in Fracture Research, Transactions 5th International Congress on Fracture, Cannes, France, Vol. 5, pp. 2163-2170.

34. T. Nishioka and S.N. Atluri, "Analysis of a Propagating Central Crack in a Finite Plate" in Proceedings International Conference on Analytical and Experimental Fracture Mechanics, Rome, Italy, July 1980.

35. T. Nishioka and S.N. Atluri, "Efficient Computational Techniques for the Analysis of Fracture in Pressure Vessels and Piping", ASME PVP-80-37, ASME Century 2 Technology Conference, San Fransisco, 12 pp., 1980.

36. T. Nishioka and S.N. Atluri, "Numerical Modeling of Dynamic Crack Propagation of Finite Bodies, Part II: Results", ASME Journal of Applied Mechanics, Vol. 47, No. 3, pp. 577-583, 1980.

37. T. Nishioka and S.N. Atluri, "Numerical Modeling of Dynamic Crack Propagation in Finite Bodies, Part I: Formulation", ASME Journal of Applied Mechanics, Vol. 47, No. 3, pp. 570-577, 1980.

38. S.N. Atluri, M. Nakagaki, and T. Nishioka, "Static/Dynamic Crack-Growth: Moving Singular Elements", Proceedings 3rd Engineering Mechanics Specialty Conference, ASCE, Austin, Texas, pp. 370-372, 1979, invited presentation.

39. S.N. Atluri, T. Nishioka, and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, by Moving Singular Elements" in Nonlinear and Dynamic Fracture Mechanics (Eds: N. Perrone and S.N. Atluri), AMD Vol. 35, ASME, NY, pp. 37-67, 1979.

40. S.N. Atluri, "Path-Independent Integrals in Finite Elasticity and Inelasticity, with Body Forces, Inertia, and Arbitrary Crack-Face Conditions", Engineering Fracture Mechanics, Vol. 16, No. 3, pp. 341-364, 1982.

41. R.B. Stonesifer and S.N. Atluri, "On a Study of the $(\Delta \underline{T})_c$ and C* Integrals for Fracture Analysis under Non-Steady Creep", Engineering Fracture Mechanics, Vol. 16, No. 5, pp. 625-643, 1982.

42. R.B. Stonesifer and S.N. Atluri, "Moving Singularity Creep Crack Growth Analysis with the $(\Delta \underline{T})_c$ and C* Integrals", Engineering Fracture Mechanics, Vol. 16, No. 6, pp. 769-782, 1982.

43. M. Nakagaki and S.N. Atluri, "A Study of the Use of the $(\dot{\underline{T}})$ Integral for Fracture of Materials with Inelastic Constitutive Laws", ASME Preprint, presented and published at ASME Winter Annual Meeting, Phoenix, AZ, November 1982.

44. M. Nakagaki and S.N. Atluri, "FEM Ductile Fracture Analysis with Emphasis on a Certain New Path Integral", Symposium on Advances and Trends in Structural & Solid Mechanics, GWU and NASA, Washington, D.C., Oct. 4-7, 1982.

45. S.N. Atluri, "On Some New General and Complementary Energy Theorems for the Rate Problems in Finite Strain, Classical Elastoplasticity", Journal of Structural Mechanics, Vol. 8, No. 1, pp. 61-92, 1980.

46. S.N. Atluri, "On Rate Principles in Finite Strain Analysis of Elastic and Inelastic Nonlinear Solids" in Recent Research on Mechanical Behavior of Solids, University of Tokyo Press, pp. 79-107, 1979.

47. S.N. Atluri and H. Murakawa, "New General and Complementary Energy Theorems, Finite Strain, Rate Sensitive Inelasticity and Finite Elements: Some Computational Studies" in Nonlinear Finite Element Analysis in Structural Mechanics (Eds: Wunderlich, Stein, and Bathe), Springer-Verlag, Berlin, pp. 28-48, 1981.

48. S.N. Atluri, "Rate Complementary Energy Principles: Finite Strain Plasticity Problems; and Finite Elements", Invited paper, in Variational Methods in Mechanics, Proceedings IUTAM Symposium on Variational Methods in Mechanics (Ed: S. Nemat-Nasser), Pergamon Press, pp. 363-368, 1981.

49. H. Murakawa and S.N. Atluri, "Finite Element Solutions of Finite Strain Elastic-Plastic Problems, Based on a New Complementary Energy Rate Principle" in Advances in Computer Methods for Partial Differential Equations (Eds: R. Vishnevetsky and B. Stepleman), IMAS, Rutgers University, pp. 53-61, 1979.

50. H. Murakawa and S.N. Atluri, "Necking Analysis of a Perfect Rectangular Bar in Plane-Strain", invited paper, 16th Annual Meeting of the Society for Engineering Science, Northwestern University, September 1979 (extended abstract only).

51. S.N. Atluri and H. Murakawa, "Consistent Assumed Stress Finite Element Methods for Finite-Strain Elasto-Plastic Analyses", invited presentation, 2nd International Conference on Computational Methods in Nonlinear Mechanics, TICOM, Austin, Texas, 1979 (extended abstract only).

52. R. Rubenstein and S.N. Atluri, "Algorithms for Integration of Objective Constitutive Relations over Finite Time Steps", Computer Methods in Applied Mechanics and Engineering, 1982.

53. S.N. Atluri, P. Tong, and H. Murakawa, "Recent Studies in Hybrid and Mixed Fem in Continuum Mechanics" in Hybrid and Mixed FEM (Eds: S.N. Atluri, R.H. Gallager, and O.C. Zienkiewicz), John Wiley and Sons, 1982.

54. K.W. Reed and S.N. Atluri, "Finite Deformation Viscoplasticity Analysis by Using Assumed Stress Finite Elements", Proceedings Symposium on Recent Develqpments in Structural Analysis, ASME-Aerospace Division, ASME WAM, Washington, D.C., ASME-AD-01, pp. 211-221, November 1981.

55. H. Murakawa and S.N. Atluri, "A Consistently Formulated Assumed Stress Hybrid FEM for Finite Strain Analysis" in Proceedings International Conference on Finite Element Methods in Nonlinear Problems, Roorkee, India, pp. 129-135, 1979.

56. N. Fukuchi and S.N. Atluri, "Finite Deformation Analysis of Shells: A Complementary Energy-Hybrid Method" in Nonlinear Finite Element Analysis of Shells, ASME AMD, Vol. 48 (Eds: T.J.R. Hughes et al.), ASME, pp. 223-249, 1981.

II. Proposed Research and Planned Approaches:

In the following a brief discussion of proposed research and planned approaches is given.

(a) Further Theoretical and Computational Studies and "Application-Phase Analyses" Using (T^*) and (ΔT^*) Parameters in Elastic-Plastic Stable Crack Growth:

As discussed in [1], the well-known J-integral, which is experimentally determined from the area under the load-deformation curves of cracked specimens, is a valid crack-tip parameter in elastic-plastic bodies only under: (i) isothermal conditions, (ii) material homogeneity, (iii) monotonic and proportional loading, (iv) no unloading, (v) no body forces and inertia, (vi) only up to the initiation of quasistatic crack growth under monotonic loading, and (vii) perhaps for very small amounts of crack growth. Under

Final E-20-697

THEORETICAL AND COMPUTATIONAL APPROACHES

IN INELASTIC AND DYNAMIC FRACTURE MECHANICS

A Research Proposal Submitted for the
Renewal of
ONR Contract No. N00014-78-C-0636

Office of Naval Research
Navy Dept., Code 474
800 N. Quincy Street
Arlington, VA 22217
Attn: Dr. Y. Rajapakse

S.N. Atluri, Regents' Professor
Center for the Advancement of Computational Mechanics
School of Civil Engineering
Georgia Institute of Technology, Atlanta, GA 30332
(404) 894-2758

Requested Funding Period: 9/1/85 - 12/1/87

	9/1/85 9/1/86	9/1/86 9/1/87	9/1/87 12/1/87
Funds Requested from ONR:	91948	97532	24766
GIT Matching Funds:	11891	13226	3234
Total	103839	110758	28000

GEORGIA TECH RESEARCH CORPORATION

GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GEORGIA 30332-0420

Telex: 542507 GTRCOCAATL
Fax: (404) 894-3120

Phone: (404) 894 4817

Refer to: RDS/02.103.000.85.012

12 March 1985

PROPOSAL NO. SCEP-85-534

Office of Naval Research
800 North Quincy Street
Arlington, Virginia 22217

Attention: Dr. Y. Rajapakse
Code 474

Subject: Research Proposal Entitled, "Theoretical and Computational Approaches in Inelastic and Dynamic Fracture Mechanics" (Renewal of Contract No. N00014-78-C-0636)

Gentlemen:

The GEORGIA TECH RESEARCH CORPORATION desires to submit for your consideration the subject proposal prepared by Dr. S. N. Atluri, Regents' Professor, Center for the Advancement of Computational Mechanics, School of Civil Engineering, Georgia Institute of Technology.

A description of the research program, the time required and estimated costs are included in the proposal. Should additional information be desired, please do not hesitate to contact Dr. Atluri at 404/894-2758 regarding technical matters or the undersigned at 404/894-4817 for administrative concerns.

In the event of an award, we propose that the work be authorized by a modification to Contract No. N00014-78-C-0636 drawn in the name of the GEORGIA TECH RESEARCH CORPORATION.

We appreciate the opportunity of submitting this proposal and look forward to the possibility of working with you on this project.

Sincerely,

Robert D. Simpkins

Robert D. Simpkins
Contracting Officer

RDS/baj
Addressee: Four (4) copies
Enclosure: Proposal - Four (4) copies

Preface:

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In current practice, the J-integral is commonly used to determine the resistance of plastically deformed structures to continued crack growth. This approach has been shown, during current research, to be valid only for monotonic loading and small amounts of crack growth; yet in the engineering community the J-resistance curve approach is believed to be conservative for all general loadings. On the other hand, the T* parameter is formulated to be theoretically valid under a wide range of loading/unloading conditions. These two fracture criteria (T* and J), along with CTOA (crack-tip opening angle), were examined for their validity and predictive capability in situations of crack growth after a cycle of loading, unloading to zero load, followed by reloading. The results of a careful, combined numerical/experimental study showed that the T* parameter accurately predicted the behavior (crack growth to begin only after 50% loading in the reloading phase), while the other parameters (J and CTOA) were seriously anti-conservative. In another study, the relevance of the parameters $\dot{T}^*$, C^*, and $\dot{T}_c$ in characterizing creep crack growth was examined. Experimental data on creep crack growth in a 316 stainless steel single-edge notch specimen was numerically simulated, and the variations of various parameters during crack growth were ascertained. The results were found conclusively in favor of the $\dot{T}^*$ parameter in characterizing creep crack growth under non-steady creep (not pure power law creep) as well as in situations wherein time-independent plastic strains are significant in addition to creep strains. These results are also significant in the contest of current and continued studies on crack propagation in viscoplastic materials.

Comprehensive studies have been conducted concerning (i) possible crack-tip parameters that may govern elasto-dyamic crack propagation in the presence of non-uniform temperature fields (caused by high-intensity energy sources), material inhomogeneity, etc.; (ii) crack-tip parameters in elasto-plastic dynamic fracture; and (iii) crack-tip parameters in rate-sensitive elastic-plastic solids. In this process, several parameters J' (elasto-dynamic crack propagation), ΔT^* (incremental parameters governing crack propagation in rate-independent or rate-dependent elasto-plastic solids), have been established to be theoretically valid.

The path-independent integral J'_k, which has the meaning of energy release rate in elasto-dynamic crack propagation, has been used numerically to obtain the mixed mode dynamic stress-intensity factors for a crack propagating in a prescribed direction with a prescribed velocity. Moving isoparametric (non-singular) elements are used to model crack propagation. Even though $\underline{J}'$ is a vector integral and hence is coordinate invariant, the desirability of using specific coordinate systems to improve the accuracies of the numerical solutions for K-factors is pointed out. Two procedures for extracting the mixed mode K-factors from the $\underline{J}'$ integral for a propagating crack are given. It is found that the component of $\underline{J}'$ along the crack axis, i.e. J'^{o} is always equal to or greater than the product of a crack-velocity function and the component normal to the crack-axis, J'^{o}. Comprehensive results were obtained to validate these procedures for engineering applications.

A fundamental study concerning the adiabatic heating due to plastic energy dissipation near a rapidly propagating crack in structural steel has been conducted. The transient temperature field (as seen by a moving observer) due to heat generated in the moving process-zone near a dynamically propagating crack-tip was analyzed by a moving-mesh finite element procedure. The effects of temperature-dependent material properties, and the loss of heat to the surrounding medium through convection and radiation are studied. Situations under which conditions in the process zone may be labelled as 'isothermal' or 'adiabatic' have been clearly delineated. Precise estimates of temperature rise near the tips of cracks propagating in structural steels have been obtained. The effects of this temperature rise on the process of fracture itself is a subject of continued inquiry.

To account for the phenomena of unloading near a propagating crack-tip, and to account for material behavior near the crack-tip under arbitrary far-field loading histories (of loading/unloading/reloading, etc.), the present computational procedures have been systematically enhanced with more realistic constitutive relations. As by-products of these efforts, certain unifying concepts underlying the internal-time, general internal variable, and multi-yield-surface theories of plasticity have been found. Rate forms of evolution equations for stress, back stress, and other variables based on a new internal-time theory have been developed. It has been shown that by an appropriate choice of kernels in the presently developed theory, the so-called "nonlinear-kinematic-hardening" rules of Chaboche, Mroz, et al., the multiple-yield-surface theories of Mroz, Krieg, Dafalias, etc., and the linear kinematic hardening theories of Prager, etc., may all be deduced as special cases. Computational algorithms for the implementation of this new theory have been systematically implemented. It has been demonstrated that while the present theory is no more difficult to implement computationally than the usual Prager-Ziegler theories, it leads to far better constitutive modeling of observed phenomenological behavior in plasticity.

Computational algorithms for numerical modeling of elastic as well as inelastic dynamic fracture have been comprehensively studied. Node-release techniques as well as mesh-translation techniques and time-integration schemes have been analyzed as to convergence and stability. Especially for rate-sensitive plastic behavior, the limits on the time-step in the time integration procedure, as dictated by the constitutive model itself, the stability of the time integration and the required 'smoothness' of the node-release (or equivalently the mesh-shift) have been systematically explored.

While the above represents a brief summary of some of the salient findings of the current research, a comprehensive discussion of the above as well as other findings of research is given in the following publications in the archival literature:

1. S.N. Atluri, M. Nakagaki, T. Nishioka, and Z-b. Kuang, "Crack-Tip Parameters and Temperature Rise in Dynamic Crack Propagation", Engineering Fracture Mechanics (Special Issue in Honor of A.S. Kobayashi; Eds: M.F. Kanninen, S.N. Atluri, Y. Rajapakse, and W.G. Knauss) (In Press).

2. F.W. Brust, J.J. McGowan, and S.N. Atluri, "A Combined Numerical/Experimental Study of Ductile Crack Growth After a Large Unloading, Using T*, J, and CTOA Approaches", Engineering Fracture Mechanics (In Press).

3. F.W. Brust, T. Nishioka, and S.N. Atluri, "Further Studies on Elastic-Plastic Stable Fracture Utilizing the T* Integral", Engineering Fracture Mechanics (In Press).

4. F.W. Brust and S.N. Atluri, "Studies on Creep Crack Growth Using the T* Integral", Engineering Fracture Mechanics (In Press).

5. Z-b. Kuang and S.N. Atluri, "Temperature Field Due to a Moving Heat Source: A Moving Mesh Finite Element Analysis", Journal of Applied Mechanics, Paper No. 85-APM-19, ASME (In Press).

6. S.N. Atluri and T. Nishioka, "Numerical Studies in Dynamic Fracture Mechanics", International Journal of Fracture (Special 20th anniversary Volume Issue on Dynamic Fracture) (In Press).

7. O. Watanabe and S.N. Atluri, "A New Endochronic Approach to Computational Elastoplasticity: Example of a Cyclically Loaded Cracked Plate", Journal of Applied Mechanics, ASME (In Press).

8. O. Watanabe and S.N. Atluri, "Internal Time, General Internal Variable and Multi-Yield-Surface Theories of Plasticity and Creep: A Unification of Concepts", International Journal of Plasticity, Pergamon Press (In Press).

9. S.N. Atluri, T. Nishioka, and M. Nakagaki, "Incremental Path-Independent Integrals in Inelastic and Dynamic Fracture Mechanics", Engineering Fracture Mechanics, Vol. 20, No. 2, pp. 209-244, 1984.

10. S.N. Atluri, T. Nishioka, and M. Nakagaki, "Recent Studies of Energy Integrals and Their Applications", Plenary Lecture, 6th International Congress on Fracture, New Delhi, India, 2-10 December 1984.

11. S.N. Atluri and T. Nishioka, "On Path-Independent Integrals in the Mechanics of Elastic, Inelastic, and Dynamic Fracture", Invited Paper, Special Issue of Honor of Professor George Irwin, Journal of Aeronautical Society of India, May 1984, 52 pp.

12. S.N. Atluri and T. Nishioka, "Computational and Theoretical Studies on Dynamic Fracture Mechanics and Three-Dimensional Crack Problems", Invited General Lecture and in Proceedings ICF International Symposium on Fracture Mechanics, Science Press, Beijing, December 1983, pp. 1-17.

13. T. Nishioka and s.N. Atluri, "On the Computation of Mixed-Mode K-Factors for a Dynamically Propagating Crack Using Path-Independent Integrals J'", Engineering Fracture Mechanics, Vol. 20, No. 2, pp. 193-208, 1984.

14. S.N. Atluri and A.S. Kobayashi, "Elastic-Plastic Fracture Mechanics", Chapter 3 in Computational Methods in the Mechanics of Fracture (Ed: S.N. Atluri), North-Holland Publishing Co. (To Appear).

15. S.N. Atluri, "Energetic Approaches and Path-Independent Integrals", Chapter 5 in Computational Methods in the Mechanics of Fracture (Ed: S.N. Atluri), North-Holland Publishing Co. (To Appear).

16. T. Nishioka and S.N. Atluri, "Computational Methods in the Mechanics of Dynamic Fracture", Chapter 8 in Computational Methods in the Mechanics of Fracture (Ed: S.N. Atluri), North-Holland Publishing Co. (To Appear).

17. S.N. Atluri, "Energy Release Rates and Path-Independent Integrals in Dynamic Fracture" in Material Behaviour Under High Stress and Ultrahigh Loading Rates (Eds: J. Mescall and V. Weiss), Plenum Press, NY, pp. 211-222, 1983.

18. S.N. Atluri, "Computational Solid Mechanics: Its Present Status and future Direction", General Lecture, 4th International Conference on Applied Mathematical Modeling, Tainan, Taiwan, December 1984.

19. M. Nakagaki, S.N. Atluri, and T. Nishioka, "On the Path-Independent Integral T_p in Elastic-Plastic Fracture", Developments in Mechanics, Vol. 12 (Proceedings SECTAM XII, Pine Mountain, GA), May 1984.

20. S.N. Atluri, "Crack-Tip Parameters in Elastic-Plastic and Dynamic Fracture", 2nd Navy Workshop on Fracture, Johns Hopkins University, Applied Physics Lab, May 1984.

21. T. Nishioka and s.N. Atluri, "On the Use of ($\underline{T}$) Integral in Visco-Plastic Dynamic Crack Propagation", Symposium on Recent Advances in Computational Mechanics, Joint ASME/ASCE Mechanics Conference, Albuquerque, NM, June 1985 (also to appear in Comp. Meth. Appl. Mech. & Engg.).

22. M. Nakagaki and S.N. Atluri, "Path-Independent Integrals Accounting for Thermomechanical Behavior Near Dynamically Propagating Cracks", Symposium on Recent Advances in Computational Mechanics, Joint ASME/ASCE Mechanics Conference, Albuquerque, NM, June 1985 (also to appear in Comp. Meth. Appl. Mech. & Engg.).

23. S.N. Atluri and T. Nishioka, "Energy Release Rates and Path-Invariant Integrals in Dynamic Fracture: Some Theoretical Developments and Computational Studies", Proceedings 4th International Conference on Fracture Mechanics in Engineering & Technology (Eds: G.C. Sih and R. Jones), Melbourne, 1982.

24. T. Nishioka and S.N. Atluri, "A Numerical Study of the Use of Path Independent Integrals in Elasto-Dynamic Crack Propagation", Engineering Fracture Mechanics, Vol. 18, No. 1, pp. 23-33, 1983.

25. T. Nishioka and S.N. Atluri, "Path-Independent Integrals, Energy Release Rates, and General Solutions of Near-Tip Fields in Mixed-Mode Dynamic Fracture Mechanics", Engineering Fracture Mechanics, Vol. 18, No. 1, pp. 1-22, 1983.

26. S.N. Atluri, "Current Studies in Inelastic, Dynamic, and Three-Dimensional Fracture Mechanics Analyses", Fracture Tolerance Evaluation (Eds: K. Kanazawa, A.S. Kobayashi, and K. Iida), Tokyo Printing Co., Japan, pp. 45-57, 1982.

27. T. Nishioka and S.N. Atluri, "Finite Element Simulations of Problems in Dynamic Fracture Mechanics", Transaction Japanese Society of Mechanical Engineers, JSME, pp. 138-145, 1982.

28. T. Niskioka and S.N. Atluri, "An Analysis of, and Some Observations on, Dynamic Fracture in an Impact Test Specimen,", Paper No. 81-PVP-18, presented at Joint Conference on Pressure Vessels and Piping, Materials, Nuclear Energy, and Solar Divisions, Denver, CO, June 1981; also in Journal of Pressure Vessel Technology.

29. T. Nishioka and S.N. Atluri, "A Method for Determining Dynamic Stress Intensity Factors from COD Measurement at the Notch Mouth in Dynamic Tear Testing", Engineering Fracture Mechanics, Vol. 16, No. 3, pp. 333-339, 1982.

30. T. Nishioka and S.N. Atluri, "Finite Element Simulation of Fast Fracture in Steel DCB Specimen", Engineering Fracture Mechanics, Vol. 16, No. 2, pp. 157-175, 1982.

31. T. Nishioka and S.N. Atluri, "Numerical Analysis of Dynamic Crack Propagation: Generation and Prediction Studies", Engineering Fracture Mechanics, Vol. 16, No. 3, pp. 303-332, 1982.

32. T. Nishioka, R.B. Stonesifer, and S.N. Atluri, "An Evaluation of Several Moving Singularity Finite Element Models for Fast Fracture Analyses", Engineering Fracture Mechanics, Vol. 15, No. 1-2, pp. 205-218, 1981.

33. S.N. Atluri and T. Nishioka, "Dynamic Fracture Analysis: A Translating Singularity Finite Element Procedure" in Advances in Fracture Research, Transactions 5th International Congress on Fracture, Cannes, France, Vol. 5, pp. 2163-2170.

34. T. Nishioka and S.N. Atluri, "Analysis of a Propagating Central Crack in a Finite Plate" in Proceedings International Conference on Analytical and Experimental Fracture Mechanics, Rome, Italy, July 1980.

35. T. Nishioka and S.N. Atluri, "Efficient Computational Techniques for the Analysis of Fracture in Pressure Vessels and Piping", ASME PVP-80-37, ASME Century 2 Technology Conference, San Fransisco, 12 pp., 1980.

36. T. Nishioka and S.N. Atluri, "Numerical Modeling of Dynamic Crack Propagation of Finite Bodies, Part II: Results", ASME Journal of Applied Mechanics, Vol. 47, No. 3, pp. 577-583, 1980.

37. T. Nishioka and S.N. Atluri, "Numerical Modeling of Dynamic Crack Propagation in Finite Bodies, Part I: Formulation", ASME Journal of Applied Mechanics, Vol. 47, No. 3, pp. 570-577, 1980.

38. S.N. Atluri, M. Nakagaki, and T. Nishioka, "Static/Dynamic Crack-Growth: Moving Singular Elements", Proceedings 3rd Engineering Mechanics Specialty Conference, ASCE, Austin, Texas, pp. 370-372, 1979, invited presentation.

39. S.N. Atluri, T. Nishioka, and M. Nakagaki, "Numerical Modeling of Dynamic and Nonlinear Crack Propagation in Finite Bodies, by Moving Singular Elements" in Nonlinear and Dynamic Fracture Mechanics (Eds: N. Perrone and S.N. Atluri), AMD Vol. 35, ASME, NY, pp. 37-67, 1979.

40. S.N. Atluri, "Path-Independent Integrals in Finite Elasticity and Inelasticity, with Body Forces, Inertia, and Arbitrary Crack-Face Conditions", Engineering Fracture Mechanics, Vol. 16, No. 3, pp. 341-364, 1982.

41. R.B. Stonesifer and S.N. Atluri, "On a Study of the $(\Delta \underline{T})_c$ and C* Integrals for Fracture Analysis under Non-Steady Creep", Engineering Fracture Mechanics, Vol. 16, No. 5, pp. 625-643, 1982.

42. R.B. Stonesifer and S.N. Atluri, "Moving Singularity Creep Crack Growth Analysis with the $(\Delta \underline{T})_c$ and C* Integrals", Engineering Fracture Mechanics, Vol. 16, No. 6, pp. 769-782, 1982.

43. M. Nakagaki and S.N. Atluri, "A Study of the Use of the $(\dot{\underline{T}})$ Integral for Fracture of Materials with Inelastic Constitutive Laws", ASME Preprint, presented and published at ASME Winter Annual Meeting, Phoenix, AZ, November 1982.

44. M. Nakagaki and S.N. Atluri, "FEM Ductile Fracture Analysis with Emphasis on a Certain New Path Integral", Symposium on Advances and Trends in Structural & Solid Mechanics, GWU and NASA, Washington, D.C., Oct. 4-7, 1982.

45. S.N. Atluri, "On Some New General and Complementary Energy Theorems for the Rate Problems in Finite Strain, Classical Elastoplasticity", Journal of Structural Mechanics, Vol. 8, No. 1, pp. 61-92, 1980.

46. S.N. Atluri, "On Rate Principles in Finite Strain Analysis of Elastic and Inelastic Nonlinear Solids" in Recent Research on Mechanical Behavior of Solids, University of Tokyo Press, pp. 79-107, 1979.

47. S.N. Atluri and H. Murakawa, "New General and Complementary Energy Theorems, Finite Strain, Rate Sensitive Inelasticity and Finite Elements: Some Computational Studies" in Nonlinear Finite Element Analysis in Structural Mechanics (Eds: Wunderlich, Stein, and Bathe), Springer-Verlag, Berlin, pp. 28-48, 1981.

48. S.N. Atluri, "Rate Complementary Energy Principles: Finite Strain Plasticity Problems; and Finite Elements", Invited paper, in Variational Methods in Mechanics, Proceedings IUTAM Symposium on Variational Methods in Mechanics (Ed: S. Nemat-Nasser), Pergamon Press, pp. 363-368, 1981.

49. H. Murakawa and S.N. Atluri, "Finite Element Solutions of Finite Strain Elastic-Plastic Problems, Based on a New Complementary Energy Rate Principle" in Advances in Computer Methods for Partial Differential Equations (Eds: R. Vishnevetsky and B. Stepleman), IMAS, Rutgers University, pp. 53-61, 1979.

50. H. Murakawa and S.N. Atluri, "Necking Analysis of a Perfect Rectangular Bar in Plane-Strain", invited paper, 16th Annual Meeting of the Society for Engineering Science, Northwestern University, September 1979 (extended abstract only).

51. S.N. Atluri and H. Murakawa, "Consistent Assumed Stress Finite Element Methods for Finite-Strain Elasto-Plastic Analyses", invited presentation, 2nd International Conference on Computational Methods in Nonlinear Mechanics, TICOM, Austin, Texas, 1979 (extended abstract only).

52. R. Rubenstein and S.N. Atluri, "Algorithms for Integration of Objective Constitutive Relations over Finite Time Steps", Computer Methods in Applied Mechanics and Engineering, 1982.

53. S.N. Atluri, P. Tong, and H. Murakawa, "Recent Studies in Hybrid and Mixed Fem in Continuum Mechanics" in Hybrid and Mixed FEM (Eds: S.N. Atluri, R.H. Gallager, and O.C. Zienkiewicz), John Wiley and Sons, 1982.

54. K.W. Reed and S.N. Atluri, "Finite Deformation Viscoplasticity Analysis by Using Assumed Stress Finite Elements", Proceedings Symposium on Recent Developments in Structural Analysis, ASME-Aerospace Division, ASME WAM, Washington, D.C., ASME-AD-01, pp. 211-221, November 1981.

55. H. Murakawa and S.N. Atluri, "A Consistently Formulated Assumed Stress Hybrid FEM for Finite Strain Analysis" in Proceedings International Conference on Finite Element Methods in Nonlinear Problems, Roorkee, India, pp. 129-135, 1979.

56. N. Fukuchi and S.N. Atluri, "Finite Deformation Analysis of Shells: A Complementary Energy-Hybrid Method" in Nonlinear Finite Element Analysis of Shells, ASME AMD, Vol. 48 (Eds: T.J.R. Hughes et al.), ASME, pp. 223-249, 1981.

II. Proposed Research and Planned Approaches:

In the following a brief discussion of proposed research and planned approaches is given.

(a) Further Theoretical and Computational Studies and "Application-Phase Analyses" Using (T^*) and (ΔT^*) Parameters in Elastic-Plastic Stable Crack Growth:

As discussed in [1], the well-known J-integral, which is experimentally determined from the area under the load-deformation curves of cracked specimens, is a valid crack-tip parameter in elastic-plastic bodies only under: (i) isothermal conditions, (ii) material homogeneity, (iii) monotonic and proportional loading, (iv) no unloading, (v) no body forces and inertia, (vi) only up to the initiation of quasistatic crack growth under monotonic loading, and (vii) perhaps for very small amounts of crack growth. Under these limiting conditions, when J is valid, however, J has been theoretically established in literature to be uniquely related to the crack-tip asymptotic fields.

To arrive at fracture crieria that may be theoretically valid in the context of flow theories of plasticity, studies under current ONR-sponsored research were aimed, as discussed in Part I [see Refs. 1-4], at crack-tip parameters T^* (and its increment ΔT^*). These crack-tip parameters can be evaluated conveniently through their equivalent far-field path-independent integral representations, which involve finite domain integrals in addition to contour integrals. The incremental crack-tip parameter ΔT^*, which is more fundamental as well as more appropriate from the point of view of computational mechanics, is defined such that: (i) it involves the incremental stress-work in the integral over Γ_ε such that it can be defined appropriately for any material model; (ii) the far-field definition of ΔT^* is inherently path-independent but involves a domain integral; (iii) it is easily defined for non-isothermal and non-homogeneous material conditions; (iv) it is a path-independent integral type crack-tip parameter even for arbitrarily large amounts of crack growth and general non-steady conditions; (v) it is a valid crack-tip parameter for arbitrary histories of loading and unloading; and (vi) when specialized to the case of "steady-state" crack propagation, the finite domain integral in the far-field representation of ΔT^* (and T^* itself) vanishes indentically. By "steady-state", it is meant that the stress-strain fields everywhere in the solid (asymptotically near the crack-tip as well as in the far-field) must remain invariant as seen by an observer moving with the crack-tip. However, this situation occurs rarely in a finite-sized cracked body with a moderate amount of crack-tip plasticity.

The work accomplished so far, as briefly discussed in Part I, clearly demonstrates the validity of T^* as a fracture parameter governing large amounts of stable crack-growth in elastic-plastic solids with moderate to large-scale crack-tip plasticity. Most of the work so far has been in performing "generation-phase" calculations (i.e. analytically simulating the experimentally measured crack-length history and external loading) to ascertain the variation of T* during sustained stable crack-growth. It was found, for instance, that while (i) J continues to rise monotonically at a rapid rate during stable crack-growth, and (ii) CTOA starts out being a large value, decreases, and levels off to a constant value during sustained growth, the new parameter T* has the desirable property that it increases (and is equal to J) during small amounts of growth but levels off to a constant value during sustained stable growth. Thus, T* has features similar to the currently widely used (combined J-CTOA) criterion, except that T* resistance curve can be unambiguously used to predict crack-growth, while the switch from J to CTOA criterion is ambiguous at best.

However, there are further theoretical issues that should be addressed before the parameter T* can be more firmly established as an engineering ductile fracture parameter. These are subjects of proposed research and are briefly addressed in the following. (i) As mentioned earlier, during sustained stable crack growth, J increases monotonically, while T* levels off to a constant value. J is a far-field parameter, while T* is a crack-tip parameter. The variation of T* is obtained currently from a detailed numerical analysis by accurately accounting for the processes in the plastic zone. To enhance the utility of T* as a simple engineering ductile fracture parameter, it would be of interest to develop estimation procedures whereby T* may be estimated without recourse to detailed numerical analysis. Currently, simple estimation procedures do exist for estimation of the far-field parameter J. The object of the proposed research is to construct simple models of the plastic zone whereby the crack-tip parameter T* may be interpolated from the estimated far-field parameter J. It appears from current research that (T*/J) is an exponential function of the crack growth $(a-a_o)$. The object of the proposed estimation procedure is to establish firmly the nature of this function. (ii) During the course of current research, the crack-tip parameter T* (and its increment ΔT^*) were directly computed on a path Γ_ε (which is of radius ε around the crack-tip and proceeds along the crack-face at a distance ε from the crack-face into the wake left behind the advancing crack-tip), using the definition of T* as given in [2-5]. Further, it should be emphasized that these computations were performed for general "non-steady" conditions. By this we mean that the stress-strain fields everywhere in the cracked specimen (i.e. asymptotically near the crack-tip as well as elsewhere in the specimen) are not the same at all times as seen by an observer moving with the crack-tip. This is typical of situations of stable fracture in a laboratory test specimen such as a compact tension specimen. However, it would be desirable to establish a theoretical relation between T* and the near-tip asymptotic solution for a crack growing in an elastic-plastic solid. To do this, a knowledge of the steady as well as non-steady state asymptotic solutions near a growing crack in a hardening material is necessary. While some progress has been made in recent years, such a complete asymptotic solution yet remains elusive for the practically important problem of mode I crack growth in plane strain/plane stress. For an ideally plastic material, the asymptotic crack-tip fields in a mode I problem has been recently studied by Slepyan [6], Gao [7], and Rice et al. [8]. Later, Rice refined these solutions [9]. While the solutions in [9] are valid during non-steady as well as state-state growth, the other solutions [6-8] are valid only for steady-state growth. Finally, Gao [10] has developed an asymptotic solution for steady-state growth in a power-law hardening material; however, it was later noted by Gao [11] that there is a deficiency in these solutions, namely that the plastic part of the strain rate does not vanish as θ (the polar coordinate centered at the crack-tip) approaches the boundary between the plastic loading and elastic unloading sectors. Thus, the issue of asymptotic fields near the crack-tip in mode I growth in a strain-hardening material needs to be clarified and is proposed to be attempted in the coming moths. Once such solutions are clarified, a unique relationship between the crack-opening displacement rate and ΔT^* can be established for strain-hardening materials, as has been developed for the ideally plastic case between $\dot{\delta}$ and J[9]. Also, a clarification of the asymptotic crack-tip fields for strain hardening materials during crack-growth would also serve to enhance the current computational procedures inasmuch as special crack-tip elements in which these singularities are embedded, may be developed. (iii) Another issue concerns the size of the path Γ_{ε}, as defined by the small parameter ε, that is necessary. Currently, ε has been chosen based on computational considerations — i.e. when the crack is grown, say, by a node-release technique, the solution within the two elements immediately adjacent to the crack-tip is not of acceptable accuracy; hence, ε is chosen so to avoid these two elements. However, more theoretically, ε should be well within the region dominated by a singular crack-tip field. While the chosen ε in current computations is always within the range of dominance of all known crack-tip solutions as described above, more rigorous criteria for T* dominance of the crack-tip fields from initiation through the transient period up to stable crack growth are necessary and are proposed to be pursued. These studies will thus complement the earlier studies in literature [12-13] concerning the J-dominance of crack-tip fields in monotonically loaded solids with stationary cracks. In the proposed research, a finite deformation analysis will be employed to numerically determine, using the necessary mesh refinements, the details of the crack-tip fields as accurately as possible. By comparing these to the analytical asymptotic solutions, criteria for T* dominance are proposed to be established.

Another item of proposed research concerns the "application-phase" studies of stable crack-growth based on the (T* vs Δa) resistance curve. Thus, using the material property of (T* vs Δa) as obtained from the 'generation-phase' calculations as described earlier, predictions will be made of crack propagation histories, and comparisons will be made with experimental data. Successful comparisons of this type in various geometries and various load conditions will ultimately establish the engineering validity of ductile fracture criteria based on T*. It should be remarked that sufficiently reliable experimental data is not easily available in open literature. The one 'application-phase' study performed so far [14] concerns the prediction of crack growth during a cycle of loading, unloading to zero load, followed by relaoding. As has been remarked earlier, T* alone accurately predicts the experimentally observed behavior of crack growth to begin at only 50% of reloading. The experimental data were made available through the courtesy of Dr. J.J. McGowan of the Oak Ridge National Laboratories. The collaboration with Dr. McGowan is continuing, and more varied experimental data for different specimen geometries and different load histories are being and will continue to be obtained. It is proposed to perform "application-phase" studies on compact-tension specimens of three different geometries and three different load conditions, to establish the geometry and load-history independence of proposed criteria based on T*.

(b) Viscoplatic Analysis of Fast Dynamic Crack Propagation and the Use of the T* Inegral:

Under realistic speeds of crack propagation, the plastic strain rate at the tip of a propagating crack can be very high, of the order of 10^3/sec to 10^5/sec. The fundamental issues germane to an understanding of the physical processes and the ultimate objective of predicting dynamic crack propagation and possible arrest at moderate to high dynamic load levels are as follows: (i) an accurate constitutive modeling of high strain-rate plasticity under conditions of "steady-state" as well as "non-steady-state" near the crack-tip; (ii) accounting for yielding conditions that may range from the classical small-scale yielding to moderate-scale yielding near the crack-tip; (iii) proper treatment of a variety of situations ranging from (α) a more practical one wherein the stress-strain fields asymptotically close to the propagating crack-tip, as well as elsewhere in the solid are "non-steady" (i.e. not the same at all times as seen by an observer moving with the crack-tip), and (β) a more idealized one wherein the asymptotic fields as well as those elsewhere in the solid are 'steady' (i.e. the same at all times as seen by an observer moving with the crack-tip), which is less likely to occur in the practical problem of a crack propagating in a finite plate; (iv) the development of proper computational algorithms with competing requirements on the time-step size used in the time-integration scheme. Specifically, the time increment used should be such that (a) the consitutitve equation for high-strain rate plasticity can be properly integrated through stable "forward-gradient' schemes, (b) the algorithm for simulation of the propagating crack-tip, viz., the node-release or alternatively the mesh-translation techniques should not induce spurious oscillations in the obtained stress/strain response, and (c) the wave propagation phenomena are properly accounted for; (v) the development of fracture criteria based on crack-tip parameters that remain valid in situations ranging from "non-steady-state" dynamic crack propagation in a finite body as described earlier to the more idealized case of "steady-state" crack propagation are described earlier; and (vi) the establishment of the relationship between the crack-tip parameters and the asymptotic crack-tip fields in viscoplastic crack propagation.

Freund and Hutchinson [15] recently presented an interesting study concerning viscoplastic effects on dynamic crack propagation under the classical assumption of small-schale yielding. In [15] fully steady-state conditions, i.e. the invariance of asymptotic as well as far-field stress/strain fields with respect of an observer moving with the crack-tip, were invoked. Also, an <u>approximate</u> analysis which is meant to capture the qualitative features of the effects of plasticity on high-speed crack-propagation we presented [15]. At strain-rates of interest, ($10^3 \sim 10^5$/sec), the plastic strain-rate was assumed to be linearly proportional to the increase in stress beyond a threshold level, τ_t. Also in [15], a path-independent integral, I, which does not involve a finite-domain integral, was identified for the considered idealized problem of "fully steady-state" propagation as described above. From this concept, the far-field stress-intensity values to propagate the crack-tip at a given velocity, under the postulate that the near-tip value of I is always to be maintained at a crack-<u>velocity-independent</u> critical value, were obtained. On the other hand, Hahn and his colleagues [16] present a finite element analysis of viscoplastic dynamic crack propagation. They employ [16] a viscoplastic constitutive law due to Perzyna, with $\dot{\varepsilon}_p$ varying as $[(\sigma/\sigma_o) - 1]^p$ where σ_o is a constant, and $p \gg 1$. Using the criterion of crack-opening displacement or CTOA vs (Δa) relation obtained from quasistatic stable growth experiments, crack propagation was studied at various velocities. It was found in [16] that the stresses did not reach a "steady-state" distribution even after substantial amounts of crack-growth. Also, it appears in [16] that all the crack-tip variables such as stress, etc., suffer from spurious oscillations of somewhat large magnitude, as the crack propagates. It was surmised [16] that these oscillations may be due to the algorithm employed in [16] for modeling crack growth.

As seen from the current state of viscoplastic dynamic crack-propagation analyses, it appears that a number of fundamental problems remain to be resolved through the use of refined computational procedures. Those that are proposed for ongoing research are briefly discussed in the following. (i) A review of literature on high-strain plasticity, as summarized for instance in [17], reveals an increasing evidence of a marked increase in flow stress with increasing strain rate. In addition, elasto-viscoplastic constitutive models that include strain-hardening may also be necessary. In this connection, a generalization of the rate-independent constitutive laws based on an endochronic concept, as developed during the course of current research [18,19], to the rate-sensitive case will be pursued. The endochronic (internal) time in this case will depend not only on the inelastic strain but also Newtonian time. Since computational algorithms for stress-integration have already been developed based on the theory in [19] and found to be easy and efficient to implement, the generalization of these algorithms to the rate-sensitive case should be a relatively simple and easy task. Also, other constitutive relations as proposed in recent literature by Naghdi [29] and others will also be considered as candidates for computational implementation. (ii) The computational procedure will account for clasical "small-scale yielding" situations as well as those where the plastic zone is moderately large. (iii) The computational procedure will be first applied to the specialized problem of "fully steady-state" dynamic crack propagation (as defined earlier) to confirm the validity of the approximate solution presented in [15] for various crack Mach numbers. To simulate "fully steady-state" conditions, as discussed in detail by Nishioka and Atluri [20], it is sufficient to translate the entire mesh representing the far-field along with the crack-tip by properly accounting for convective terms. The object of this study is twofold: to obtain the ratio (G/G^c_{tip}) (a) when (G^c_{tip}) is a velocity-independent constant, and (b) when G^c_{tip} is prescribed to be a function of crack-velocity. (v) The computational algorithm will be applied to study the more important practical problem of transient, fully "non-steady" (as defined earlier) dynamic crack propagation in a viscoplastic solid when the crack-tip plasticity is not restricted by the "small-scale" yielding assumption. Here, the general numerical techniques, as detailed in Nishioka and Atluri [20] for modeling dynamic crack propagation, will be useful. The computational model will be used in its "generation-phase" mode, i.e. the experimental data on crack propagation in test specimen, i.e. boundary and initial conditions, and crack length (and velocity) history will be inputs to the computational model. The object of this study is to determine the T* parameter (as defined in [4] for dynamic crack-propagation in general rate-dependent solids) as a function of crack velocity history. [The validity of T* in general fully "non-steady" crack propagation is discussed later]. By repeating these calculations for various specimens in which cracks have been initiated at various initial apparent remote K-factors, the material property, T* vs a (crack velocity) may be ascertained. In this connection, the experimental data of Kalthoff and others, as well as those of Joyce at the Naval Academy, will be used. (iv) For the constitutive relation chosen for the present viscoplastic problem, a "forward gradient" algorithm as detailed in Atluri and Murakawa [21] and Reed and Atluri [22] will be developed for implementation in finite element procedure as well as in stress-point integration. The "forward gradient" method allows a "reasonably" large time-increment and yet retains stability. Also, simulation of transient crack-propagation will be carried out by either (a) translating only the elements immediately surrounding the crack-tip by an arbitrary amount that is not equal to the finite element mesh-spacing or (b) by releasing the nodal forces in a nonlinear fashion, wherein the nodal force holding the crack-tip at its current location will be reduced to zero as a nonlinear function of the time taken by the crack-tip to travel from the current crack-tip location to the next. Thus, an "increment" of crack-tip force is removed in a "sub-increment" of time. These algorithms should alleviate the problem of large spurious oscillations noted by Hahn et al. [16]. (v) It can be shown easily that the path-independent-integral crack-tip parameter T^*, as defined for dynamic crack propagation in rate-dependent solids by Atluri et al. [2-4], is such that the "finite-domain integral" in its far-field definition vanishes identically under the idealized case of "fully steady-state" conditions. Thus, under "fully steady-state" conditions, T^* possesses the same features as the parameter "I" introduced by Freund and Hutchinson [15]. However, unlike I, the crack-tip parameter T^* is defined for fully "non-steady-state" dynamic crack propagation. Thus, T^* is anticipated to be a useful parameter in characterizing transient dynamic crack propagation in viscoplastic solids. Thus, as discussed under research item (iii) above, the present research aims at obtaining (T^* vs $\dot{a}$) relation to characterize dynamic crack propagation. (vi) The computational procedure is proposed to be used in its "application-phase" (as defined earlier) to <u>predict</u> dynamic crack propagation and arrest, in mode I conditions, in test specimens and compare such <u>predictions</u> with experimental data. (vii) The numerical results for crack-tip stress/strain data obtained in the course of the proposed research can be used to address the question of dominance of asymptotic fields [23-25] in dynamic crack propagation.

(c) Effects of Crack-Tip Temperature Rise and Finite Deformations on Dynamic Crack Propagation:

Recently a comprehensive study [26] of the temperature field near the propagating crack-tip due to heat generated through energy dissipation in the plastic zone that propagates along with the crack-tip has been conducted, wherein: (i) the heat produced per unit volume (Q) in the plastic zone per unit of crack growth has been assumed to be a constant, independent of crack velocity; (ii) the spatial distribution of Q, in the propagating plastic zone, is assumed, alternatively, to be either uniform or to possess a $(1/\sqrt{r})$ type singularity; (iii) the solution is presented for the transient temperature field as seen by an observer moving with the crack-tip at the same velocity; (iv) the temperature dependence of thermal conductivity and specific heat are accounted for; and (v) the effects of convective and radiative heat transfer to the surrounding medium are accounted for. The problem, mathematically, is a strongly nonlinear moving-boundary problem, which has been solved [26] by a moving-mesh finite element procedure. For small velocities of crack-propagation, the process near the crack-tip is nearly isothermal, i.e. heat diffuses quickly away into the remainder of the cracked solid; but, for fast, realistic speeds of crack-propagation, the process is nearly adiabatic, i.e. all the heat is dissipated in the process zone near the crack-tip — thus leading to a substantial increase of temperature near the crack-tip. Furthermore, the temperature gradients near the crack-tip are very severe. It has also been found [26] that for low velocities, the maximum temperature occurs near the center of the propagating plastic zone, while at higher velocities the maximum temperature occurs in the wake of the heat source. The calculations in [26] show that at realistic crack-propagation speeds in structural steels, while the front of the propagating process-zone remains cold, higher temperatures (and severe temperature gradients) may persist in and behind the process zone even at distances of the order of the size of the process zone.

The influence of the above-mentioned temperature rise and severe temperature gradients on the mechanical problem of crack-tip stress-strain fields in elastic-plastic fast fracture is proposed to be investigated via a coupled thermo-plastic formulation. Both small-scale yielding conditions and moderately large yield conditions near the crack-tip will be postulated. The proposed studies would aid in assessing the quantitative effects of the crack-tip temperature field on the experimental measurement of crack-tip parameters, such as the dynamic stress-intensity factor, for example through the method of shadow optics. The method of shadow optics or caustics depends on the thickness changes near the crack-tip and relates the caustic diameter to the dynamic stress intensity factor. However, in reducing the measured caustics data to dynamic stress intensity factor, the current practice is to use simple <u>isothermal</u> plane-stress solutions for crack-tip fields in linear elastic solids and the attendant thickness change near the crack-tip. The proposed research thus will provide a more realistic theoretical basis for this experimental data reduction. A formulation of the coupled thermo-plastic phenomena in the process zone and the attendant numerical studies are proposed. Guidance from earlier literature on appropriate constitutive relations for use in thermoplasticity, as for instance in [27-29], will be sought in developing the proposed computational procedures.

Another nonlinear phenomenon in the process zone, whose effects on dynamic crack propagation have so far not been explored, pertain to finite deformations in the process zone. Ductile fracture is in general characterized by processes such as void growth and void coalescence occuring near the crack-tip. These processes are primarily of a finte strain type, which are also manifest in the phenomenon of crack-tip blunting. So far, all analyses of fast crack propagation have been of a small-strain type. With a view towards gaining a basic understanding of the process-zone behavior in inelastic cynamic crack propagation, a finite strain analysis is proposed. However, consideration of finite strain effects shall be confined to a small region near the crack-tip. Since refined experimental methods being used by current experimentalists yield precise data on finite deformations near the crack-tip, these will provide a much-needed comparison for the computational models.

(e) More Precise Comparison of Present Computational Models with Experimental Data in Fast Crack Propagation:

Experimental work on fast fracture has so far been based mostly on dynamic photoelasticity or the method of shadow optics. Experimental data that has been reported widely was primarily restricted to the stress-intensity factor variation with time and crack-length versus time histories. Computational methods developed currently under ONR support gave results that are in excellent agreement with these reported experimental data. However, for a more precise understanding of dynamic fracture and crack arrest, a closer computational simulation of experimental data appears necessary. Currently, experimental work is underway by other ONR investigators, such as F-P. Cheng, based on Moire techniques and holography, which may produce data that is much more refined than available hitherto. These data may include crack-surface deformation profiles, finite deformation effects near the crack-tip, etc. It is proposed that the accuracy of the currently developed computational techniques, as well as those being proposed, be verified in comparison to these refined experimental data.

It is anticipated, however, that this aspect of the proposed research should constitute but a small fraction of the overall effort.

(f) Dynamic Fracture in Plates and Shells:

So far all the analytical (as well as experimental) work in dynamic fracture has been limited to plane stress or plane strain situations of inplane loading. The practical problem, however, often involves the assessment of integrity of flawed plates and shells (often stiffened) which are subject to out-of-plane dynamic impulsive loading. As a first step, an asymptotic analysis of crack-tip fields for constant velocity crack propagation in a thin plate, with through-the-thickness crack and subject to pure moments, will be undertaken using a simple fourth-order plate bending theory. Here the objective will be to obtain the asymptotic solution for transverse displacement w and bending resultants $M_{\alpha\beta}$. It is known in the static case that the fourth-order plate bending theory leads to certain inconsistencies in the boundary-conditions on the crack-face. Thus, as in the static case, the use of a sixth-order plate bending theory may be needed in the dynamic crack propagation case and will be pursued. The generalization of the path-independent integrals to the bending problem of plates and shells will be pursued. The incorporation of the local crack-tip response analysis algorithms into programs for overall structural response of stiffened plates and shells is the ultimate future objective of this item of research.

Schedule of Research:

It is anticipated that frequent contact will be maintained with cognizant ONR program officials to appraise the progress of research, to seek advice on any deviations from proposed efforts, and to pursue any topical problems that may come to light at ONR contractor review meetings, etc.

Project Budget:

It is proposed that the level of effort in the requested renewal period 9/1/85 to 12/1/87 be the same as that during the current contract period, 9/1/83-9/1/85. However, effective 7/1/85, the indirect expense rate for research contracts and grants at Georgia Institute of Technology will increase to 64.2% from the current 47.2%. (However, this increase is approved by cognizant auditing agencies of the U.S. government.) This increase in indirect expense rate is reflected in the increase in funding requested from ONR in each 12-month period beginning 9/1/85 even though the level of effort is proposed to be the same as the current.

References:

[1] Atluri, S.N., Nakagaki, M., Nishioka, T., and Kuang, Z-b., "Crack-Tip Paramenters and Temperature Rise in Dynamic Crack Propagation", Engineering Fracture Mechanics (Special issue in honor of A.S. Kobayashi; M.F. Kanninen et al., Eds.) (In press).

[2] Atluri, S.N., Nishioka, T., and Nakagaki, M., "Incremental Path-Independent Integrals in Inelastic and Dynamic Fracture Mechanics", Engineering Fracture Mechanics, Vol. 20, No. 2, pp. 209-244, 1984.

[3] Atluri, S.N., Nishioka, T., and Nakagaki, M., "Recent Studies of Energy Integrals and Their Applications" in Advances in Fracture Research, Vol. 1 (Eds: S.R. Valluri, et al.), Proceedings ICF6, Pergamon, 00. 181-210, 1984.

[4] Atluri, S.N. and Nishioka, T., "On Path-Independent Integrals in Inelastic and Dynamic Crack Propagation", J. aeronautical Society of India (Special issue in honor of G.R. Irwin), 52 pp., November 1984 (In press).

[5] Brust, F.W., Nishioka, T., and Atluri, S.N., "Further Studies on Elastic-Plastic Stable Fracture Utilizing the T* Integral", Engineering Fracture Mechanics) (In press).

[6] Slepyan, L.I., "Growing Crack During Plane Deformation of an Elastic-Plastic Body", Izve. ANSSR, Mekhanika Tverdogo Tela, Vol. 9, No. 1, pp. 57-64, 1974.

[7] Gao, Y-C. and Huang, K-C., "Elastic-Plastic Fields at Cracktips in a Perfectly Plastic Medium", Proceedings IUTAM Symposium on Three-Dimensional Constitutive Relationships and Ductile Fracture, 1980.

[8] Rice, J.R., Drugan, W.J., and Sham, T.L., "Elastic-Plastic Analysis of Growing Cracks", ASTM STP 700, pp. 198-221, 1980.

[9] Rice, J.R., "Elastic-Plastic Crack Growth", Mechanics of Solids — The R. Hill 60th Anniversary Volume (Eds: Hopkins and Sewell), Pergamon, 1982.

[10] Gao, Y-C. and Hwang, K-C., "Elastic-Plastic Fields in Steady Crack Growth in a Strain-Hardening Material", Advances in Fracture Research (Ed: D. Francois), Vol. 2, pp. 669-682, 1981.

[11] Gao, Y-C. and Hwang, K-C., "Asymptotic Near-Tip Solution for Mode III Crack in Steady Growth in Power Hardening Materials", International Journal of Fracture, Vol. 21, pp. 301-317, 1983.

[12] McMeeking, R.M. and Parks, D.M., "On Criteria for J-Dominance of Crack-Tip Fields in Large-Scale Yielding", ASTM STP 668, pp. 175-194, 1979.

[13] Parks, D.M., "The Dominance of Crack-Tip Fields of Inelastic Continuum Mechanics", Numerical Methods in Fracture Mechanics (Eds: Owen & Hinton), 1980.

[14] Brust, F.W., McGowan, J.J., and Atluri, S.N., "A Combined Numerical/Experimental Study on Ductile Crack Growth After a Large Unloading, Using T*, J, and CTOA Criteria", Engineering Fracture Mechanics (In press).

[15] Freund, L-B. and Hutchinson, J.W., "High Strain-rate Crack Growth in Rate-Dependent Plastic Solids", Mech-53, Harvard University (Presented at International Symposium on Dynamic Fracture, San Antonio, November 1984).

[16] R. Hoff, C.A. Rubin, and G.T. Hahn, "Viscoplastic Finite Element Analysis of Rapid Fracture", Presented at International Symposium on Dynamic Fracture, San Antonio, November 1984.

[17] Clifton, R.J., "Dynamic Plasticity", Journal of Applied Mechanics, Vol. 50, pp. 941-952, December 1983.

[18] Watanabe, O. and Atluri, S.N., "Internal Time, General Internal Variable, and Multi-Yield-Surface Theories of Plasticity and Creep: A Unification of Concepts", International Journal of Plasticity (In press).

[19] Watanabe, O. and Atluri, S.N., "A New Endochronic Approach to Computational Elasto-Plasticity: Example of a Cyclically Loaded Cracked Plate", Journal of Applied Mechanics (In press).

[20] Nishioka, T. and Atluri, S.N., "Computational Methods in the Mechanics of Dynamic Fracture" to appear in (Ed: S.N. Atluri) Computational Methods in the Mechanics of Fracture, North-Holland.

[21] Atluri, S.N. and Murakawa, H., "New General and Complementary Energy Theorems, Finite Strain, Rate Sensitive Inelasticity and Finite Elements: Some Computational Studies", Nonlinear Finite Element Analysis in Structural Mechanics (Eds: W. Wunderlich et al.), Springer-Verlag, pp. 28-48, 1981.

[22] Reed, K.W. and Atluri, S.N., "Analyses of Large Quasistatic Deformations of Inelastic Bodies by a New Hybrid-Stress Finite Element Algorithm", Computer Methods in Applied Mechanics and Engineering, Vol. 39, pp. 245-295, 1983.

[23] Achenbach, J.D. and Dunyavesky, V., "Fields Near a Rapidly Propagating Crack-tip in an Elastic-Perfectly-Plastic Material", J. Mech. Phys. Solids, Vol. 29, pp. 283-303, 1981.

[24] Lo, K.K., "Dynamic Crack-tip Fields in Rate-Sensitive Solids", J. Mech. Phys. Solids, Vol. 31, pp. 287-305, 1983.

[25] Gao, Y-C. and Nemat-Nasser, S., "Dynamic Fields Near a Crack-Tip Growing in an Elastic-Perfectly-Plastic Solid", Mech. of Material, Vol. 2, pp. 47-60, 1983.

[26] Kuang, Z-b. and Atluri, S.N., "Temperature Field Due to a Moving Heat Source: A Moving Mesh Finite Element Analysis", Journal of Applied Mechanics, ASME (In press).

[27] Rice, J.R., "Continuum Mechanics and Thermodynamics of Plasticity in Relation to Microscale Deformation Mechanisms" in Constitutive Relations in Plasticity (Ed: A.S. Argon), M.I.T., 1975.

[28] Green, A.E. and Naghdi, P.M., "A Thermodynamic Development of Elastic-Plastic Continua", Proceedings IUTAM Symposium on Irreversible Aspects of Continuum Mechanics & Transfer of Physical Characteristics in Moving Fluids (Eds: H. Parkus et al.), Springer, pp. 117-131, 1966.

[29] Naghdi, P.M., "Constitutive Relations for Idealized Elastic-Viscoplastic Materials", Journal of Applied Mechanics, Vol. 51, pp. 93-101, 1984.